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FIRST-ORDER THEORY FOR UNSTEADY MOTION OF THIN WINGS AT SUPERSONIC SPEEDS

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SUMMARY

An approximate method is presented for determining the pressure distribution due to unsteady motion for thin wings of fairly general plan form. The local source strength, which is proportional to the normal component of the perturbation velocity, is represented by a power series in the wing coordinates, in which the coefficients are functions of time. The method is valid for arbitrary motions when the second derivative of the perturbation velocities with respect to time is not large.

As examples of the method, the load distributions due to oscillations normal to the plane of the wing and pitching oscillations about a spanwise axis are evaluated as functions of time for a swept wing with straight supersonic leading and trailing edges. For the frequency of oscillation chosen in these examples, the magnitude of the loading differed only slightly from that obtained by neglecting the time delays, but the positions of the constant-pressure lines were noticeably altered.

INTRODUCTION

The problem of predicting the pressure distribution over thin wings undergoing unsteady motion at supersonic speeds has been studied by a number of investigators. In reference 1, equations are given for the velocity potential in the vicinity of thin wings for which the top and bottom surfaces are mutually independent. An extension to finite wings with interacting top and bottom surfaces is given in reference 2, but the area-cancellation technique required therein is shown to be valid only when the second derivative of the perturbation velocities with respect to time is not too great.

For such relatively slow motions, however, a simpler method of analysis than that presented in reference 2 can be developed.

This method, which is presented herein, consists in expressing the local source strength in a power series of the coordinates, so that integrals similar to those encountered for steady motion are obtained. This method is shown to be accurate to the same degree of approximation as that given in reference 2. The analysis was completed at the NACA Lewis laboratory during June 1949.

SYMBOLS

The following symbols are used in this report:

A	amplitude of oscillation
a,a',d,d',d'',e	integration limits
a_n,b_n	functions of time
b	wing chord
C_p	pressure coefficient
c	velocity of sound
f,g,h,i,j	functions of x, y, and t
k	slope of straight leading edge in oblique coordinates
M	Mach number
S	area included in forward Mach cone from (x,y)
t	time
U	free-stream velocity
u,v	oblique coordinates parallel to Mach lines
u_d,v_d	oblique coordinates of wing vertex
u_w,v_w	oblique coordinates of point on wing surface
w	component of perturbation velocity in z-direction
x,y,z; ξ,η	Cartesian coordinates

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z_w	vertical coordinate of any point on wing
α_0	angle of chordline with respect to free-stream direction
β	cotangent of Mach angle, $\sqrt{M^2-1}$
η_0	half-wing span
σ	effective local source strength, w/U
$\sigma_{a,b}$	source strength at time $t-\tau_a$ plus source strength at time $t-\tau_b$
τ_a	time delay, $\frac{(x-\xi)M}{\beta^2 c}+\frac{\sqrt{(x-\xi)^2-\beta^2(y-\eta)^2}}{\beta^2 c}$
τ_b	time delay, $\frac{(x-\xi)M}{\beta^2 c}-\frac{\sqrt{(x-\xi)^2-\beta^2(y-\eta)^2}}{\beta^2 c}$
φ	perturbation-velocity potential
ω	angular velocity of oscillation

ANALYSIS

For thin wings, the equation for the time-dependent velocity potential in the plane of the wing, as given in references 1 and 2, is

$$\varphi(x,y,t) = -\frac{U}{2\pi}\iint_S \frac{\sigma_{a,b}\, d\xi\, d\eta}{\sqrt{(x-\xi)^2-\beta^2(y-\eta)^2}} \tag{1}$$

where the source-strength distribution is given by

$$\sigma_{a,b} = \sigma(\xi,\eta,t-\tau_a) + \sigma(\xi,\eta,t-\tau_b) \tag{2}$$

The integration is performed in the $z=0$ plane over the area included in the forward Mach cone from any point (x,y) and may include areas off the wing surface. The time delays τ_a and τ_b are given by (reference 1)

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$$\tau_a = \frac{(x-\xi)M}{\beta^2 c} + \frac{\sqrt{(x-\xi)^2-\beta^2(y-\eta)^2}}{\beta^2 c}$$

$$\tau_b = \frac{(x-\xi)M}{\beta^2 c} - \frac{\sqrt{(x-\xi)^2-\beta^2(y-\eta)^2}}{\beta^2 c} \tag{3}$$

The functional dependence of $\sigma_{a,b}$ on these time delays can be rendered explicit by expanding the two terms of equation (2) in Taylor's series about τ_a and τ_b, respectively:

$$\sigma_{a,b} = 2\left[\sigma_0(\xi,\eta,t) - \frac{\tau_a+\tau_b}{2}\frac{\partial\sigma_0(\xi,\eta,t)}{\partial t} + \frac{1}{2!}\left(\frac{\tau_a{}^2+\tau_b{}^2}{2}\right)\frac{\partial^2\sigma_0(\xi,\eta,t)}{\partial t^2} - \ldots\right] \tag{4}$$

where $\sigma_0(\xi,\eta,t)$ is the source strength corresponding to zero time delay. This representation of the actual source-strength distribution $\sigma_{a,b}$ requires that none of the derivatives of $\sigma_0(\xi,\eta,t)$ are infinite and that the series is convergent. An estimate of the magnitude of the angular velocity, for which higher-order terms can be neglected, can be obtained by considering an oscillatory motion of the type

$$\sigma_0(\xi,\eta,t) \propto e^{i\omega t}$$

For such motions, successive derivatives of σ_0 increase in magnitude by the factor ω. If τ_a and τ_b are of the order $M/(1000\ \beta^2)$ seconds (see equation (3)), then successive terms of equation (4) decrease approximately in the ratio $\omega M/(1000\ \beta^2)$. If an error of approximately 5 percent is considered allowable in the representation of $\sigma_{a,b}$, then the neglect of all except the zero- and first-order terms may be considered valid for angular velocities less than approximately $(200\beta^2)/M$ radians per second.

In reference 3, the zero-order approximation for unsteady motion is mentioned. The solution for this approximation, as seen from equations (4) and (1), is at each instant equivalent 1214 to the steady-state solution corresponding to the instantaneous distribution of source strengths. The solution for the first-order approximation is considered herein. (Special cases for this approximation are discussed in references 4 and 5 but the derivations are not general.) The source-strength distribution for this approximation becomes

$$\sigma_{a,b} = 2\left[\sigma_0(\xi,\eta,t) - \frac{(x-\xi)M}{\beta^2 c}\frac{\partial\sigma_0(\xi,\eta,t)}{\partial t}\right] \tag{5}$$

In order to obtain solutions for finite wings with interacting upper and lower surfaces, the procedures developed by Evvard, whereby the effect of disturbed flow outboard of the wing is replaced by an equivalent integration over the wing surface, can be employed. In reference 2, this area-cancellation technique, which was originally developed for steady motion, is shown, in effect, to be valid also for unsteady motion if the motion is such that the square-root terms in the time-delay expression (equation (3)) are negligible in the equation for $\sigma_{a,b}$. Inasmuch as these square-root terms do not appear in the zero-order and the first-order approximations given in equation (5), the area-cancellation technique is entirely valid up to the first-order approximation. No general procedure is yet available for higher-order approximations when the upper and lower surfaces of the wing interact.

Equation (5) leads to an essential simplification of the procedure described in reference 2 for types of motion for which the area-cancellation technique is valid. This simplification consists in expressing the velocity potential in terms of integrals commonly encountered for the steady motion of wings. If, for example, the motion can be represented as a power series in the coordinates, such as

$$\sigma_0(\xi,\eta,t) = \sum_n \left(a_n\xi^n + b_n\eta^n\right) \tag{6}$$

then equation (5) becomes

$$\sigma_{a,b} = 2 \sum_n \left[\left(a_n - \frac{xM}{\beta^2 c} \frac{\partial a_n}{\partial t}\right) \xi^n + \frac{M}{\beta^2 c} \frac{\partial a_n}{\partial t} \xi^{n+1} + \frac{M}{\beta^2 c} \frac{\partial b_n}{\partial t} \xi \eta^n + \left(b_n - \frac{xM}{\beta^2 c} \frac{\partial b_n}{\partial t}\right) \eta^n \right] \tag{7}$$

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where a_n and b_n are any function of time consistent with the restrictions imposed by retaining only the first two terms of the Taylor's series (equation (4)). Substitution of equation (7) in equation (1) results in integrals of the type

$$\iint_S \frac{\xi^r \eta^s \, d\xi \, d\eta}{\sqrt{(x-\xi)^2 - \beta^2 (y-\eta)^2}}$$

where r and s are integers and the integrals have coefficients that are functions of x, y, and t.

Although motions may occur that require values of n greater than one in equation (6), many common types of unsteady motion are represented by values of n of zero and one. For these motions, equation (7) becomes

$$\sigma_{a,b} = f(x,y,t) + g(x,y,t)\xi + h(x,y,t)\eta + i(x,y,t)\xi\eta + j(x,y,t)\xi^2 \tag{8}$$

where

$$f = 2\left[a_0 + b_0 - \frac{xM}{\beta^2 c}\left(\frac{\partial a_0}{\partial t} + \frac{\partial b_0}{\partial t}\right)\right]$$

$$g = 2\left[a_1 - \frac{xM}{\beta^2 c}\frac{\partial a_1}{\partial t} + \frac{M}{\beta^2 c}\left(\frac{\partial a_0}{\partial t} + \frac{\partial b_0}{\partial t}\right)\right]$$

$$h = 2\left(b_1 - \frac{xM}{\beta^2 c}\frac{\partial b_1}{\partial t}\right)$$

$$i = \frac{2M}{\beta^2 c} \frac{\partial b_1}{\partial t}$$

$$j = \frac{2M}{\beta^2 c} \frac{\partial a_1}{\partial t}$$

If equation (8) is used in equation (1), the quantity f at any instant is seen to be an effective angle of attack, and Ug and Uh are effective rates of steady pitch and roll about the $\xi=0$ and $\eta=0$ axes, respectively. For a wing of the type for which the methods of reference 2 are applicable, the procedures used to determine the velocity potential in steady flight at angle of attack and in steady roll and pitch may be used for all unsteady motions representable by the first three terms of equation (8). When the velocity potential has been determined, the load distribution is obtained from the equation

$$C_p = -\frac{2}{U}\left(\frac{\partial\varphi}{\partial x} + \frac{1}{U}\frac{\partial\varphi}{\partial t}\right)$$

which, from equations (8) and (1) becomes

$$C_p = -\frac{2}{U}\left[f\frac{\partial\varphi_1}{\partial x} + g\frac{\partial\varphi_2}{\partial x} + h\frac{\partial\varphi_3}{\partial x} + i\frac{\partial\varphi_4}{\partial x} + j\frac{\partial\varphi_5}{\partial x} + \varphi_1\left(\frac{\partial f}{\partial x} + \frac{1}{U}\frac{\partial f}{\partial t}\right) + \right.$$

$$\varphi_2\left(\frac{\partial g}{\partial x} + \frac{1}{U}\frac{\partial g}{\partial t}\right) + \varphi_3\left(\frac{\partial h}{\partial x} + \frac{1}{U}\frac{\partial h}{\partial t}\right) + \varphi_4\left(\frac{\partial i}{\partial x} + \frac{1}{U}\frac{\partial i}{\partial t}\right) +$$

$$\left.\varphi_5\left(\frac{\partial j}{\partial x} + \frac{1}{U}\frac{\partial j}{\partial t}\right)\right] \tag{9}$$

where

$$\varphi_1 = -\frac{U}{2\pi}\iint_S \frac{d\xi\, d\eta}{\sqrt{(x-\xi)^2-\beta^2(y-\eta)^2}}$$

$$\varphi_2 = -\frac{U}{2\pi}\iint_S \frac{\xi\,d\xi\,d\eta}{\sqrt{(x-\xi)^2-\beta^2(y-\eta)^2}}$$

$$\varphi_3 = -\frac{U}{2\pi}\iint_S \frac{\eta\,d\xi\,d\eta}{\sqrt{(x-\xi)^2-\beta^2(y-\eta)^2}}$$

$$\varphi_4 = -\frac{U}{2\pi}\iint_S \frac{\xi\eta\,d\xi\,d\eta}{\sqrt{(x-\xi)^2-\beta^2(y-\eta)^2}}$$

$$\varphi_5 = -\frac{U}{2\pi}\iint_S \frac{\xi^2\,d\xi\,d\eta}{\sqrt{(x-\xi)^2-\beta^2(y-\eta)^2}}$$

The first term within the brackets of equation (9) yields a pressure coefficient corresponding to steady flight at angle of attack f; the second term is the pressure coefficient due to steady pitch at instantaneous rate Ug; and the third term is the pressure coefficient corresponding to steady roll at instantaneous rate Uh. Equations for these pressure coefficients are derived for a fairly general class of wings in reference 6. The remaining terms in equation (9), however, are not, for the most part, to be found in steady-motion literature and must be independently evaluated.

APPLICATION

As an illustration of the procedure outlined in the section entitled "Analysis," explicit expressions will be derived for the load distribution resulting from motions expressible in the form

$$\sigma_{a,b} = f(x,y,t) + g(x,y,t)\xi \tag{10}$$

A flat-plate wing of the plan form shown in figure 1 will be considered. This wing is similar to that analyzed in reference 6, except that no subsonic trailing edges appear.

The integrations required to determine load distribution are simplified if, as in reference 2, the equations are transformed to the oblique coordinate system whose axes are parallel to the Mach lines. The transformation equations are:

$$\left.\begin{aligned}
u &= \frac{M}{2\beta}(\xi-\beta\eta) & v &= \frac{M}{2\beta}(\xi+\beta\eta)\\
\xi &= \frac{\beta}{M}(v+u) & \eta &= \frac{1}{M}(v-u)\\
u_w &= \frac{M}{2\beta}(x-\beta y) & v_w &= \frac{M}{2\beta}(x+\beta y)\\
x &= \frac{\beta}{M}(v_w+u_w) & y &= \frac{1}{M}(v_w-u_w)
\end{aligned}\right\} \tag{11}$$

The elementary area is $(2\beta/M^2)du\,dv$. In this system of coordinates, for types of motion given by equation (10), equation (9) becomes

$$C_p = -\frac{2}{U}\left[f\,\frac{M}{2\beta}\left(\frac{\partial\varphi_1}{\partial u_w}+\frac{\partial\varphi_1}{\partial v_w}\right)+g\,\frac{M}{2\beta}\left(\frac{\partial\varphi_2}{\partial u_w}+\frac{\partial\varphi_2}{\partial v_w}\right)+\right.$$

$$\left.\varphi_1\left(\frac{\partial f}{\partial x}+\frac{1}{U}\frac{\partial f}{\partial t}\right)+\varphi_2\left(\frac{\partial g}{\partial x}+\frac{1}{U}\frac{\partial g}{\partial t}\right)\right] \tag{12}$$

where

$$\varphi_1 = -\frac{U}{2M\pi}\iint_S \frac{du\,dv}{\sqrt{(u_w-u)(v_w-v)}}$$

$$\varphi_2 = -\frac{U\beta}{2M^2\pi}\iint_S \frac{(v+u)\,du\,dv}{\sqrt{(u_w-u)(v_w-v)}}$$

The integration procedure, which employs area-cancellation technique for regions influenced by areas off the wing plan form, is the same as that described in reference 6, so that only an outline of the derivation need be repeated herein. As in reference 6, the origin of coordinates is placed at one of the junctures of supersonic and subsonic leading edges. The straight supersonic leading edges are defined by $u_1 = -kv$ and $u_2 = \frac{M\eta_0}{k}(1-k) - \frac{v}{k}$, respectively, and the curved subsonic leading edges are defined by $u = u_3(v)$ and $v = v_4(u)$, respectively. The Mach lines from the vertex and from the junctures of the supersonic and subsonic leading edges divide the plan form into eight types of region. The integration limits in each region are found as described in reference 6. The presentation is simplified if the expression for C_p is given for a general region of the wing, together with the values of the limits to be used for each region. For the general region the quantities required in equation (12) become

$$\varphi_1 = \frac{U}{M\pi}\left\{\left[\sqrt{(u_w-u)\left(v_w+\frac{u}{k}\right)}+\frac{u_w+kv_w}{\sqrt{k}}\tan^{-1}\sqrt{\frac{u_w-u}{kv_w+u}}\right]_d^a + \right.$$

$$\left[\sqrt{(u_w-u)\left(v_w+ku-M\eta_0(1-k)\right)}+\frac{ku_w+v_w-M\eta_0(1-k)}{\sqrt{k}}\tan^{-1}\sqrt{\frac{k(u_w-u)}{v_w+ku-M\eta_0(1-k)}}\right]_{d'}^{a'} -$$

$$\left. 2\sqrt{(u_w-d'')(v_w-v_4)}\right\} \tag{13}$$

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$$
\begin{aligned}
\varphi_2 = \frac{U\beta}{M^2\pi} \Bigg\{ & \Bigg[\left(\frac{2}{3} v_w + \left(1 - \frac{1}{3k}\right)\left(\frac{3u_w+kv_w}{4} + \frac{u}{2}\right)\right)\sqrt{(u_w-u)\left(v_w + \frac{u}{k}\right)} + \\
& \frac{u_w+kv_w}{\sqrt{k}}\left(\frac{2}{3} v_w + \left(1 - \frac{1}{3k}\right)\frac{3u_w-kv_w}{4}\right)\tan^{-1}\sqrt{\frac{u_w-u}{kv_w+u}}\Bigg]_d^a + \\
& \Bigg[\left(\frac{2}{3} v_w + M\eta_0 \frac{1-k}{3} + \left(1 - \frac{k}{3}\right)\left(\frac{3ku_w+v_w-M\eta_0(1-k)}{4k} + \frac{u}{2}\right)\right)\sqrt{(u_w-u)\left(v_w+ku-M\eta_0(1-k)\right)} + \\
& \frac{ku_w+v_w-M\eta_0(1-k)}{\sqrt{k}}\Bigg(\frac{2}{3} v_w + \frac{M\eta_0(1-k)}{3} + \\
& \left(1 - \frac{k}{3}\right)\frac{3ku_w-v_w+M\eta_0(1-k)}{4k}\Bigg)\tan^{-1}\sqrt{\frac{k(u_w-u)}{v_w+ku-M\eta_0(1-k)}}\Bigg]_{d'}^{a'} - \\
& 2\,\frac{2u_w+2v_w+v_4+d''}{3}\sqrt{(u_w-d'')(v_w-v_4)}\Bigg\}
\end{aligned}
\qquad (13a)
$$

$$
\begin{aligned}
\frac{M}{2\beta}\left(\frac{\partial\varphi_1}{\partial u_w} + \frac{\partial\varphi_1}{\partial v_w}\right) = \frac{U}{2\pi\beta}\Bigg\{ & -\left(1 - \frac{dv_4}{du}\right)\sqrt{\frac{u_w-d''}{v_w-v_4}} - \left(1 - \frac{du_3}{dv}\right)\sqrt{\frac{v_w-e}{u_w-u_3}} + \\
& \frac{k+1}{\sqrt{k}}\left(\left[\tan^{-1}\sqrt{\frac{u_w-u}{kv_w+u}}\right]_d^a + \left[\tan^{-1}\sqrt{\frac{k(u_w-u)}{v_w+ku-M\eta_0(1-k)}}\right]_{d'}^{a'}\right)\Bigg\}
\end{aligned}
\qquad (13b)
$$

$$
\begin{aligned}
\frac{M}{2\beta}\left(\frac{\partial\varphi_2}{\partial u_w}+\frac{\partial\varphi_2}{\partial v_w}\right) = \frac{U}{2M\pi}\Bigg\{ & \left[-\left(\frac{2}{3}\,u_w+v_4+\frac{d''}{3}\right)\left(1-\frac{dv_4}{du}\right)-4(v_w-v_4)\right]\sqrt{\frac{u_w-d''}{v_w-v_4}} - \\
& \left(\frac{2}{3}\,v_w+u_3+\frac{e}{3}\right)\left(1-\frac{du_3}{dv}\right)\sqrt{\frac{v_w-e}{u_w-u_3}} + \left(2-\frac{1-k^2}{2k}\right)\left[\sqrt{(u_w-u)\left(v_w+\frac{u}{k}\right)}\right]_d^a + \\
& \left(2+\frac{1-k^2}{2k}\right)\left[\sqrt{(u_w-u)\,\left(v_w+ku-M\eta_0(1-k)\right)}\right]_{d'}^{a'} + \\
& \left(2\,\frac{u_w+kv_w}{\sqrt{k}}-\frac{(1-k^2)(u_w-kv_w)}{2k\sqrt{k}}\right)\left[\tan^{-1}\sqrt{\frac{u_w-u}{kv_w+u}}\right]_d^a + \\
& \left(\frac{1-k^2}{2k\sqrt{k}}\left[ku_w-v_w+M\eta_0(1+k)\right]+2\,\frac{ku_w+v_w-M\eta_0(1-k)}{\sqrt{k}}\right)\left[\tan^{-1}\sqrt{\frac{k(u_w-u)}{v_w+ku-M\eta_0(1-k)}}\right]_{d'}^{a'}\Bigg\}
\end{aligned}
$$

(13c)

where the integration limits for each region are given in the following table:

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Limit	Wing regions (fig. 1)							
	I	II	III	IV	V	VI	VII	VIII
a	u_w	u_d	u_w	u_d	u_3	u_d	u_3	$-kv_4$
d	$-kv_w$	$-kv_w$	u_3	u_3	u_3	u_3	u_3	u_d
a'	u_w	u_w	u_w	u_w	u_w	$\frac{M\eta_0}{k}(1-k)-\frac{v_4}{k}$	$\frac{M\eta_0}{k}(1-k)-\frac{v_4}{k}$	u_d
d'	u_w	u_d	u_w	u_d	u_3	u_d	u_3	u_3
d"	u_w	u_w	u_w	u_w	u_w	$\frac{M\eta_0}{k}(1-k)-\frac{v_4}{k}$	$\frac{M\eta_0}{k}(1-k)-\frac{v_4}{k}$	$-kv_4$
e	v_w	v_w	$-\frac{u_3}{k}$	$-\frac{u_3}{k}$	$-ku_3 + M\eta_0(1-k)$	$-\frac{u_3}{k}$	$-ku_3 + M\eta_0(1-k)$	$-ku_3 + M\eta_0(1-k)$

Substitution of equations (13) to (13c), with the appropriate limits, in equation (12) gives the pressure coefficient in each region corresponding to unsteady motions of the type of equation (10).

From equations (13) to (13c) and (12), load distributions corresponding to two types of unsteady motion have been computed for the wing shown in figure 2. For this wing, which has streamwise tips and supersonic trailing edges parallel to the supersonic leading edges, only the first five regions of the wing shown in figure 1 are present.

The first type of motion considered is an oscillation of the wing in the z-direction at constant attitude α_0 with respect to the free-stream direction. The source strength as a function of time is

$$\sigma_0(t) = \frac{w(t)}{U} = A \cos \omega t + \alpha_0 \tag{14}$$

where A is the amplitude of oscillation. The amplitude A is a constant herein but can also be a function of time. Inasmuch as the load distribution due to steady flight at angle of attack is known and can be superimposed on the load distribution due to the oscillation, only the first term of equation (14) will be considered, that is, the motion is to be defined by 1214

$$\sigma_0(t) = A \cos \omega t = a_0(t) \tag{15}$$

Comparison of equation (15) with equations (6) and (8) shows that for this motion,

$$\sigma_{a,b} = 2A \left(\cos \omega t + \frac{xM\omega}{\beta^2 c} \sin \omega t\right) - \left(\frac{2M\omega A}{\beta^2 c} \sin \omega t\right) \xi \tag{16}$$

For the top surface of the wing, the required expressions for the coefficients and their derivatives become

$$f = -2A \left(\cos \omega t + \frac{xM\omega}{\beta^2 c} \sin \omega t\right) \tag{17}$$

$$g = \frac{2M}{\beta^2 c} A\omega \sin \omega t \tag{17a}$$

$$\frac{\partial f}{\partial x} = - \frac{2M}{\beta^2 c} A\omega \sin \omega t \tag{17b}$$

$$\frac{1}{U} \frac{\partial f}{\partial t} = \frac{2A\omega}{Mc} \left(\sin \omega t - \frac{xM\omega}{\beta^2 c} \cos \omega t\right) \tag{17c}$$

$$\frac{\partial g}{\partial x} = 0 \tag{17d}$$

$$\frac{1}{U} \frac{\partial g}{\partial t} = \frac{2A\omega^2}{\beta^2 c^2} \cos \omega t \tag{17e}$$

Substitution of equations (17) to (17e) and (13) to (13c) in equation (12) yields the desired expression for the load distribution as function of wing coordinates and time.

The second type of motion considered is an oscillation of the wing about the spanwise axis, $\xi = 0$. The appropriate expression for the source strength can be obtained by considering the equation of motion of the z coordinate of the wing. This equation is

$$z_w = A\xi \cos \omega t \tag{18}$$

At the wing surface the boundary condition is

$$\sigma_0 = \left(\frac{w}{U}\right)_{z=0} = \frac{1}{U}\frac{dz_w}{dt} = \frac{\partial z_w}{\partial \xi} + \frac{1}{U}\frac{\partial z_w}{\partial t} \tag{19}$$

so that

$$\sigma_0(\xi,t) = A \cos \omega t - \frac{A\xi\omega}{Mc}\sin \omega t = a_0(t) + a_1(t)\xi \tag{20}$$

Inasmuch as the load distribution due to the first term in equation (20) is known from the preceding example, only the equation

$$\sigma_0 = a_1(t)\xi \tag{20a}$$

need be considered here. The total load distribution for the pitching oscillation can be obtained by superposition of the load distribution due to vertical oscillation and that obtained for equation (20a). Comparison of equation (20a) with equations (6) and (8) shows that

$$\sigma_{a,b} = -\frac{2A\omega}{Mc}\left(\sin \omega t + \frac{xM\omega}{\beta^2 c}\cos \omega t\right)\xi - \left(\frac{2A\omega^2}{\beta^2 c^2}\cos \omega t\right)\xi^2 \tag{21}$$

For the top surface of the wing, the required coefficients and their derivatives are

$$g = \frac{2A\omega}{Mc}\left(\sin \omega t + \frac{xM\omega}{\beta^2 c}\cos \omega t\right) \tag{22}$$

$$j = \frac{2A\omega^2}{\beta^2 c^2}\cos \omega t \tag{22a}$$

$$\frac{\partial g}{\partial x} = \frac{2A\omega^2}{\beta^2 c^2}\cos \omega t \tag{22b}$$

$$\frac{1}{U}\frac{\partial g}{\partial t} = \frac{2A\omega^2}{M^2 c^2}\left(\cos \omega t - \frac{xM\omega}{\beta^2 c}\sin \omega t\right) \tag{22c}$$

$$\frac{1}{U}\frac{\partial j}{\partial t} = -\frac{2A\omega^3}{M\beta^2 c^3}\sin \omega t \tag{22d}$$

The remaining coefficients and derivatives in equation (9) are zero.

The unsteady load distributions for the two types of motion represented by equations (15) and (20), were computed as function of time for the wing shown in figure 3. The Mach number, angular velocity, and velocity of sound were assumed to be $\sqrt{2}$, 60 radians per second, and 1000 feet per second, respectively. For these values, terms containing ω^2/c^2 could be neglected for both types of motion with an error of less than 1 percent in the values of $\sigma_{a,b}$. This error includes the effect of retaining only the first two terms of the Taylor's series (equation (4)). The omission of terms of order ω^2/c^2 eliminates equations (17e), (22a), (22b), (22c), and (22d), and the second terms in equations (17c) and (22) from the computation. For the wing analyzed, the aspect ratio is $\frac{|\eta_0|}{b} = M = \sqrt{2}$ and the sweepback angle is 26.6° ($k = 1/3$). Nondimensional coordinates $(u_w/b,\ v_w/b)$ were used in the computation.

The results of the computations are shown in figures 4 and 5. For vertical oscillations, the loading at $t=0$ is the steady-state lift distribution corresponding to an effective angle of attack equal to the amplitude A (equation (16)). This load distribution is also obtained for the second type of motion when $t=0$ (equations (20) and (21)). For $\omega t = \pi/4$ (figs. 4(b) and 5(a)), the load distributions obtained by retaining the first-order terms in the source strengths are compared with those obtained with a zero-order approximation. The zero-order load distributions, which correspond to a neglect of the time delays, are seen to differ considerably from those obtained by first-order theory, although for the low angular velocity used the magnitudes of the loadings are almost the same. For $\omega t = \pi/2$ (figs. 4(c) and 5(b)), the loading resulting from first-order theory is seen to be small. The loading is zero by the zero-order approximation for the vertical oscillation. The load distribution for $\omega t = \pi, 5\pi/4,$ and $3\pi/2$ are the negative of the distributions for $\omega t = 0, \pi/4,$ and $\pi/2,$ respectively.

SUMMARY OF THEORY AND RESULTS

A method has been presented for determining the pressure distribution on fairly general classes of thin wings undergoing unsteady motion for which the second derivative of the perturbation velocity, with respect to time, is not large. The method consisted in expressing the local time-dependent source strengths that represent the wing motion in a power series of the coordinates, so that integrals similar to those encountered for steady motion were obtained.

For a rather general wing with no subsonic trailing edges, an explicit expression for the pressure coefficient due to unsteady motion was obtained as a function of time. As examples, the load distributions for a swept flat-plate wing, with straight supersonic leading and trailing edges and a streamwise tip, undergoing oscillations in a vertical direction and pitching oscillations about a spanwise axis, were evaluated. For the frequency of oscillation chosen, the magnitude of the loading differed only slightly from that obtained by neglecting the time delays, but the positions of the lines of constant pressure were noticeably altered.

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, August 10, 1949.

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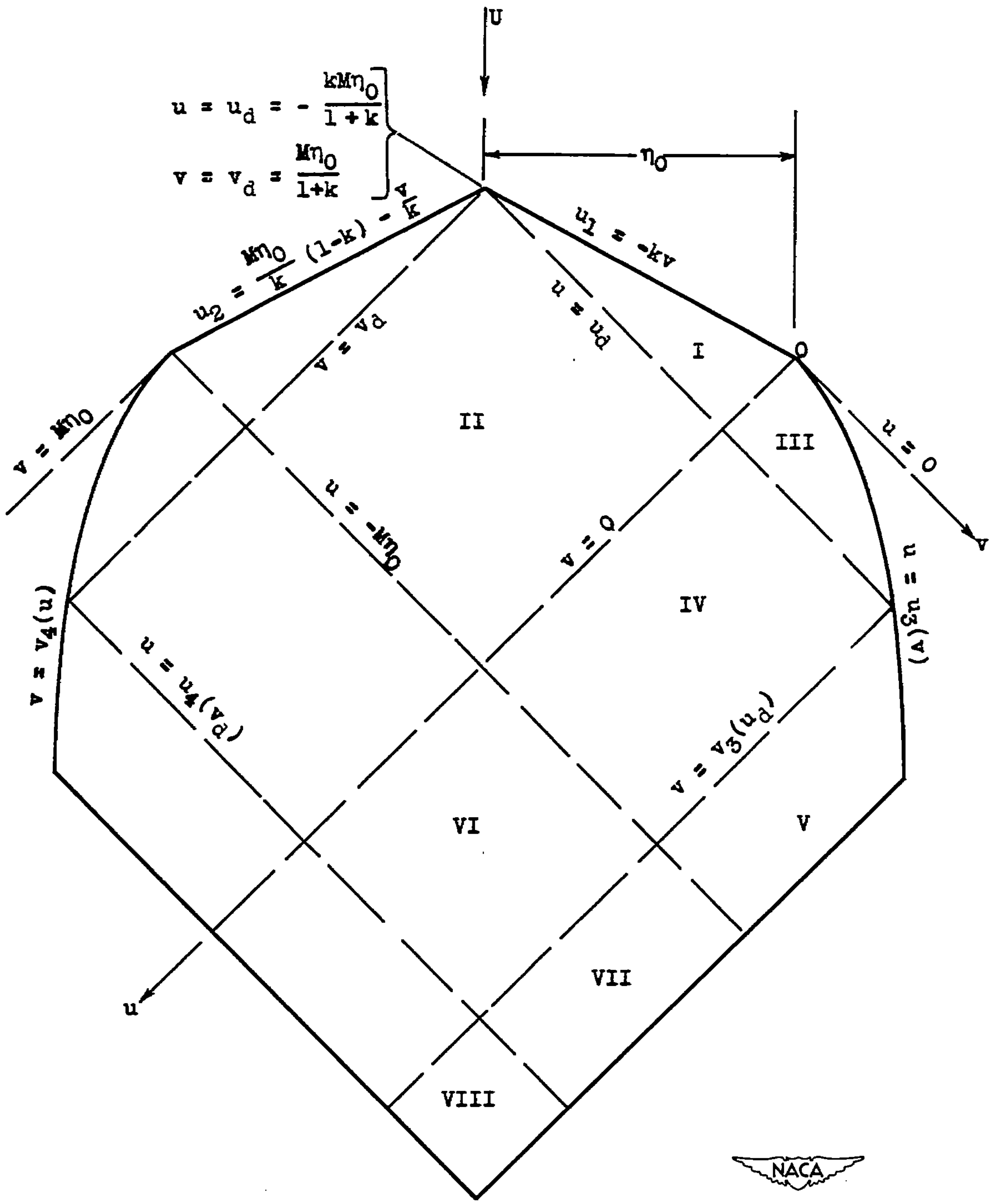

Figure 1. - Wing regions and geometrical parameters for symmetrical wing with straight supersonic leading edges, arbitrary subsonic leading edges, and trailing edges swept along Mach lines.

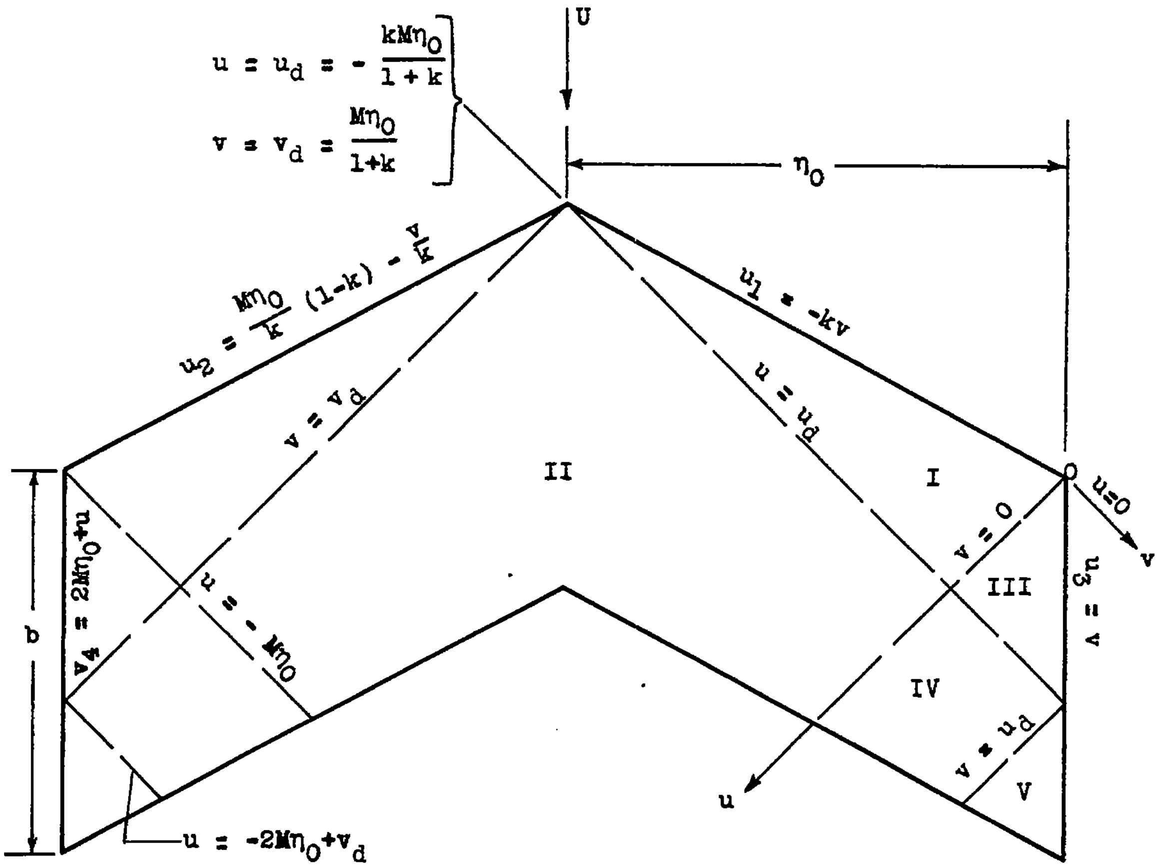

Figure 2. - Wing regions and geometric parameters for symmetrical wing with straight supersonic leading edges, trailing edges parallel to leading edges, and streamwise tips.

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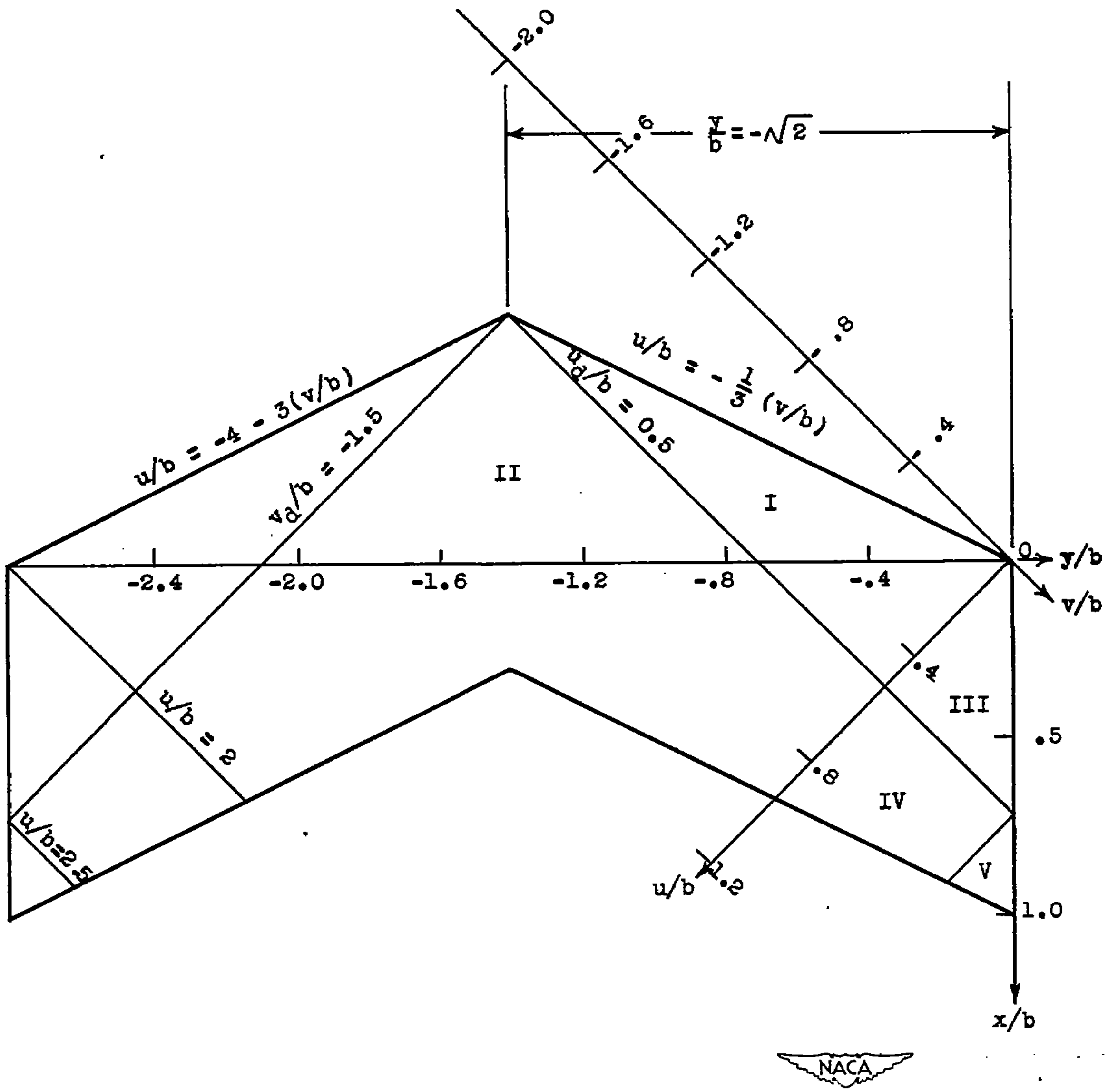

Figure 3. - Form of wing analyzed. Mach number, $\sqrt{2}$.

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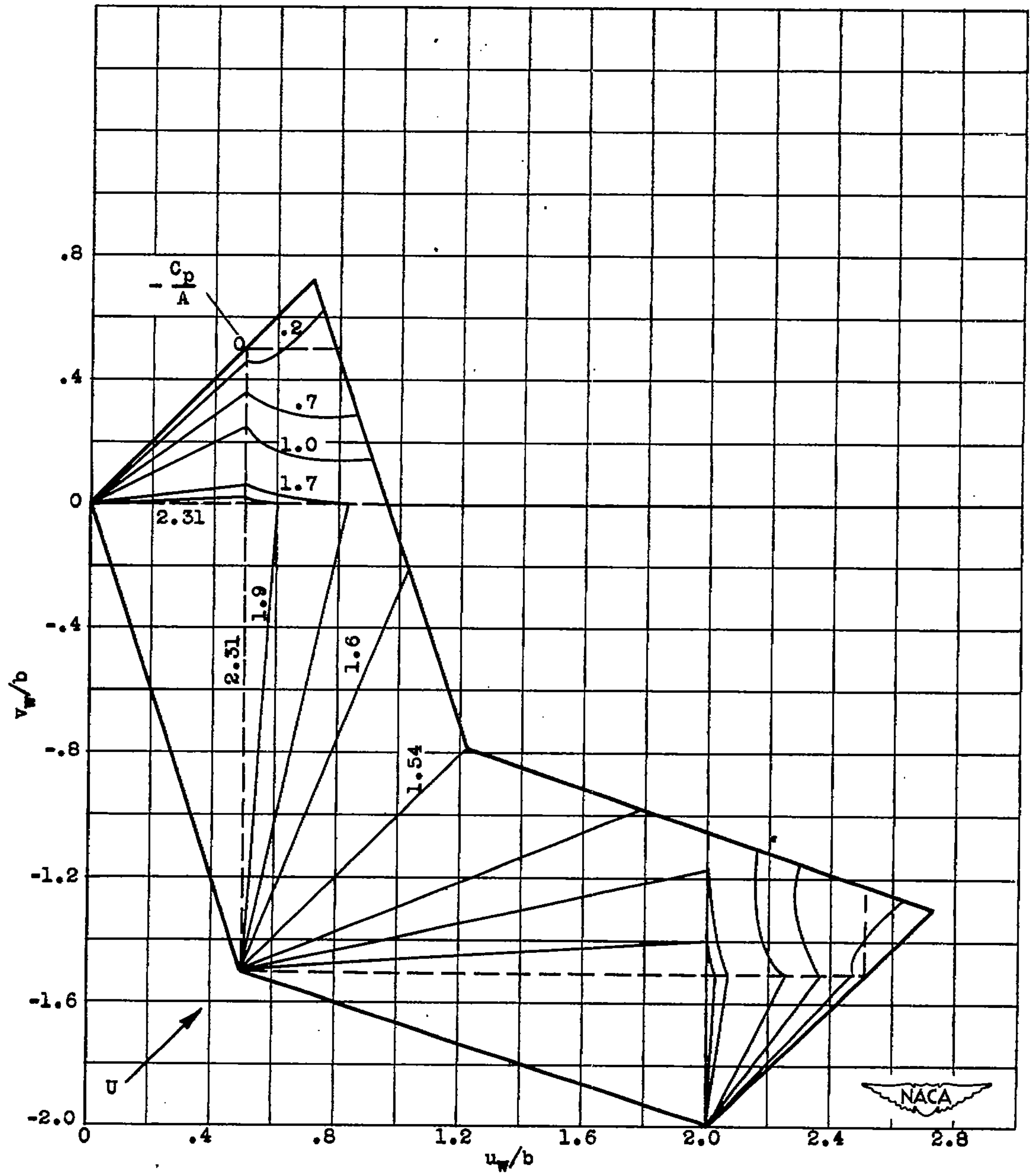
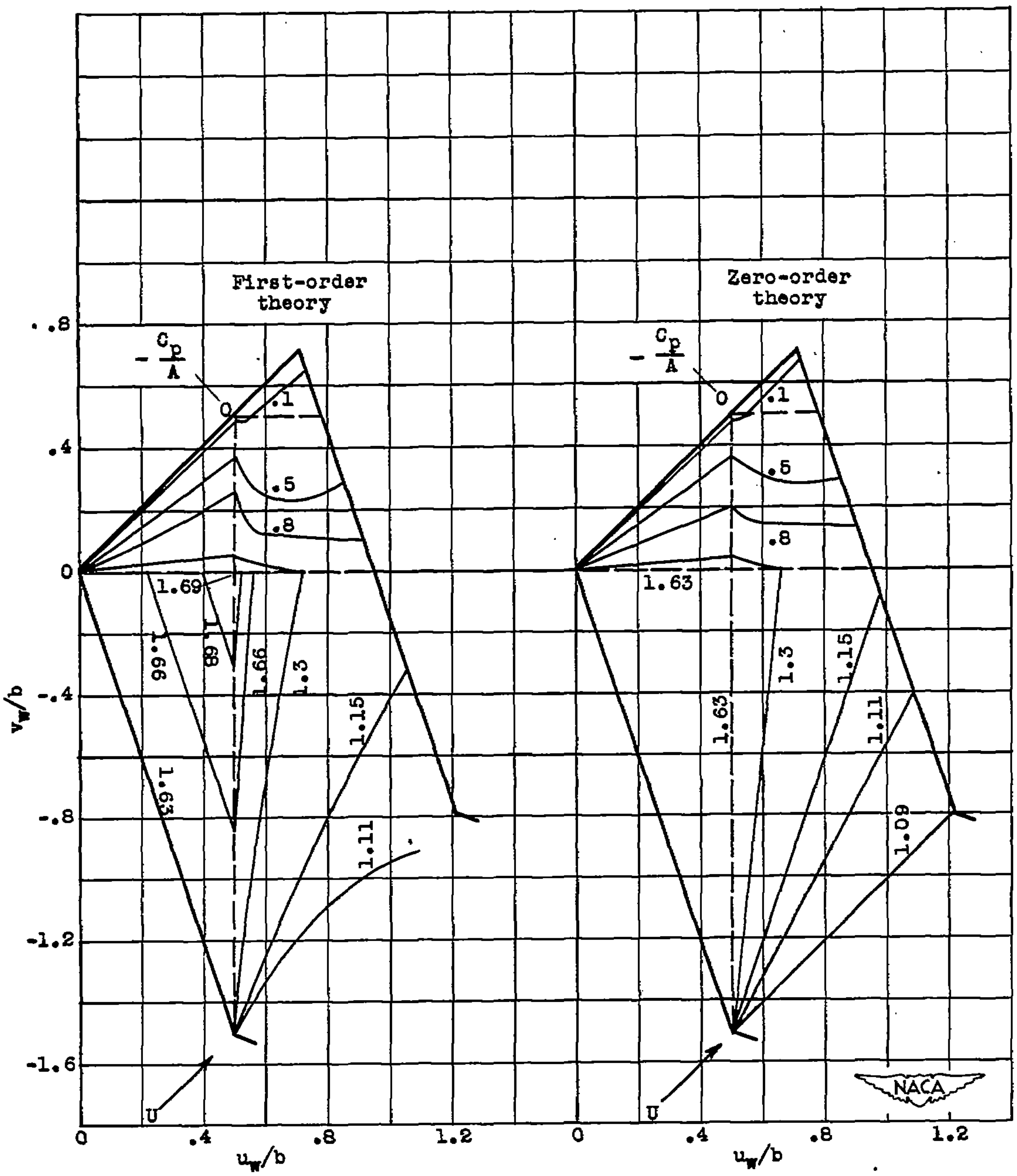


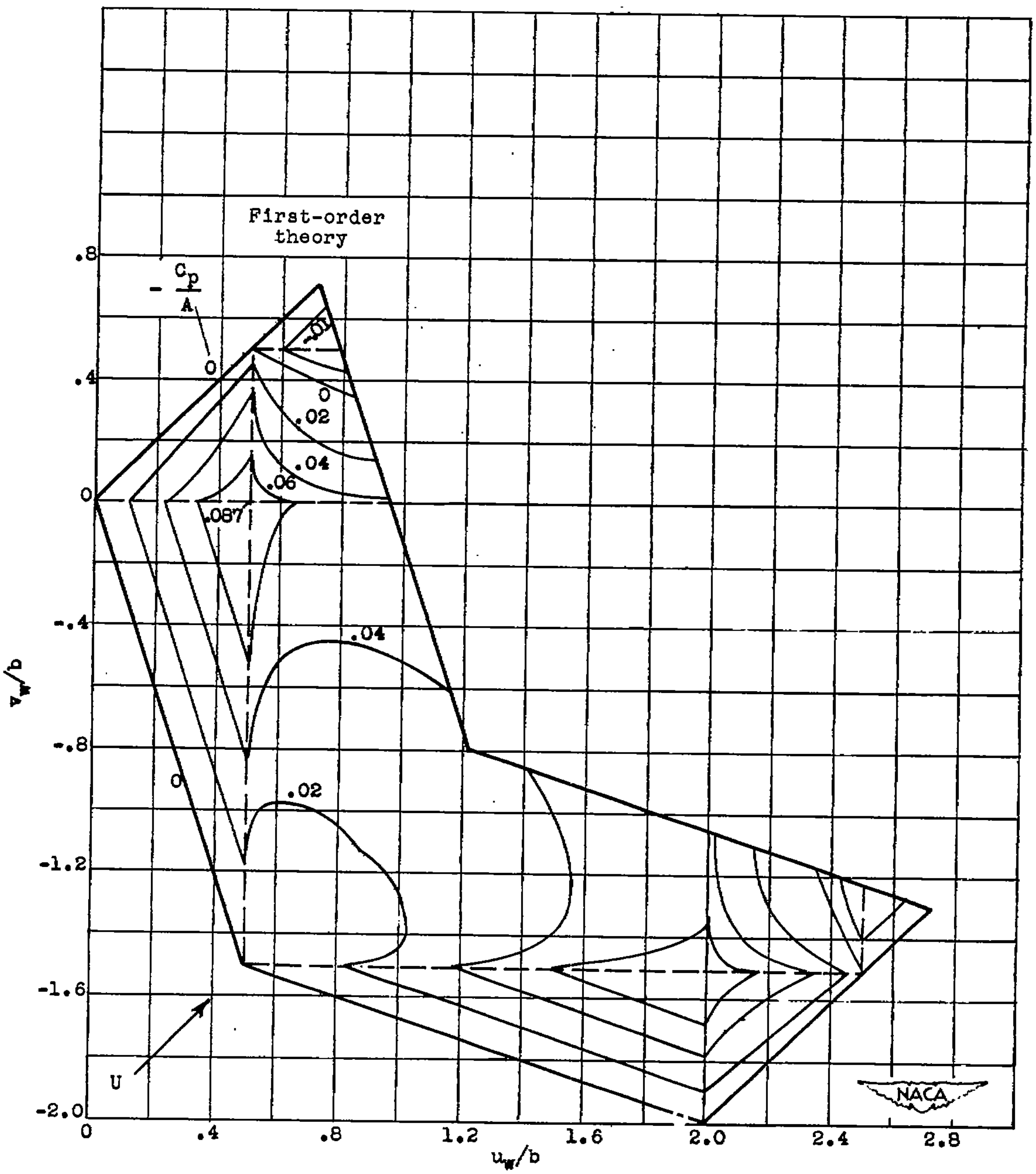
(a) $\omega t = 0$.

Figure 4. - Load distribution due to oscillations in vertical direction.



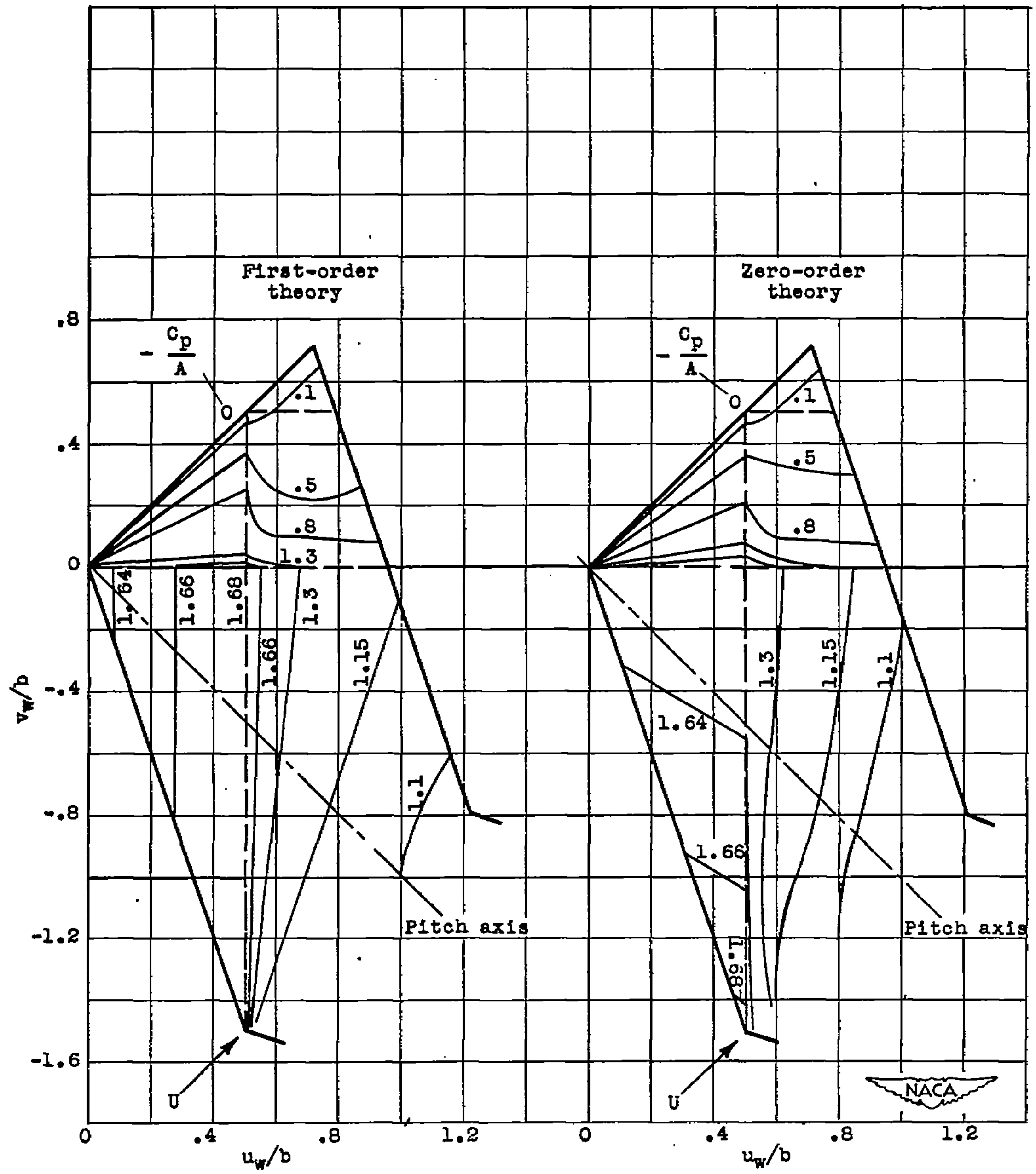
(b) $\omega t = \pi/4$.

Figure 4. - Continued. Load distribution due to oscillations in vertical direction.



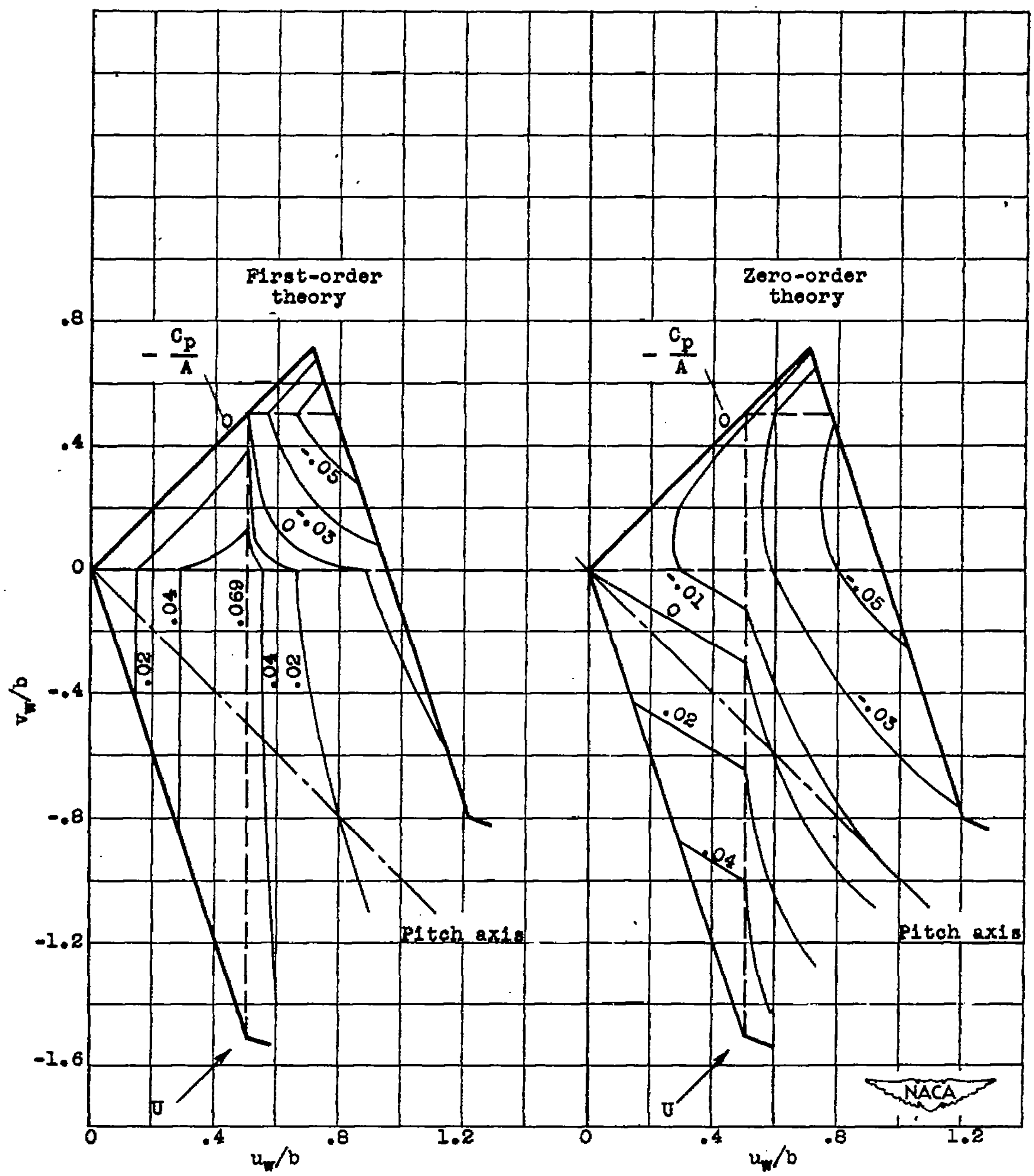
(c) $\omega t = \pi/2$.

Figure 4. - Concluded. Load distribution due to oscillations in vertical direction.



(a) $\omega t = \pi/4$.

Figure 5. - Load distribution due to pitching oscillations about y-axis.



(b) $\omega t = \pi/2$.

Figure 5. - Concluded. Load distribution due to pitching oscillations about y-axis.